HOMEWORK 11

INSTRUCTIONS

• Every learner should submit his/her own homework solutions. However, you are allowed to discuss the homework with each other (in fact, I encourage you to form groups and/or use the forums) – but everyone must submit his/her own solution; you may not copy someone else’s solution. • The homework will be peer-graded. In analytics modeling, there are often lots of different approaches that work well, and I want you to see not just your own, but also others. • The homework grading scale reflects the fact that the primary purpose of homework is learning:

Rating Meaning Point value (out of 100) 4 All correct (perhaps except a few details) with a deeper solution than expected 100 3 Most or all correct 90 2 Not correct, but a reasonable attempt 75 1 Not correct, insufficient effort 50 0 Not submitted 0

Question 15.2

In the videos, we saw the “diet problem”. (The diet problem is one of the first large-scale optimization problems to be studied in practice. Back in the 1930’s and 40’s, the Army wanted to meet the nutritional requirements of its soldiers while minimizing the cost.) In this homework you get to solve a diet problem with real data. The data is given in the file diet.xls.

**1. Formulate an optimization model (a linear program) to find the cheapest diet that satisfies the maximum and minimum daily nutrition constraints, and solve it using PuLP. Turn in your code and the solution. (The optimal solution should be a diet of air-popped popcorn, poached eggs, oranges, raw iceberg lettuce, raw celery, and frozen broccoli. UGH!)**

The titles for each piece of data can be seen below:

**cj**= cost per unit of food jj

**aij** = amount of nutrient 𝑖𝑖 per unit of food jj

**mi**= minimum amount of nutrient ii required

**Mi**= maximum amount of nutrient ii required

The variables for the problem can be seen below:

**Xj** = amount of food jj eaten

Status: Optimal  
Foods\_Celery,\_Raw = 52.64371  
Foods\_Frozen\_Broccoli = 0.25960653  
Foods\_Lettuce,Iceberg,Raw = 63.988506  
Foods\_Oranges = 2.2929389  
Foods\_Poached\_Eggs = 0.14184397  
Foods\_Popcorn,Air\_Popped = 13.869322  
{'Frozen Broccoli': Chosen\_Frozen\_Broccoli, 'Carrots,Raw': Chosen\_Carrots,Raw, 'Celery, Raw': Chosen\_Celery,\_Raw, 'Frozen Corn': Chosen\_Frozen\_Corn, 'Lettuce,Iceberg,Raw': Chosen\_Lettuce,Iceberg,Raw, 'Peppers, Sweet, Raw': Chosen\_Peppers,\_Sweet,\_Raw, 'Potatoes, Baked': Chosen\_Potatoes,\_Baked, 'Tofu': Chosen\_Tofu, 'Roasted Chicken': Chosen\_Roasted\_Chicken, 'Spaghetti W/ Sauce': Chosen\_Spaghetti\_W\_\_Sauce, 'Tomato,Red,Ripe,Raw': Chosen\_Tomato,Red,Ripe,Raw, 'Apple,Raw,W/Skin': Chosen\_Apple,Raw,W\_Skin, 'Banana': Chosen\_Banana, 'Grapes': Chosen\_Grapes, 'Kiwifruit,Raw,Fresh': Chosen\_Kiwifruit,Raw,Fresh, 'Oranges': Chosen\_Oranges, 'Bagels': Chosen\_Bagels, 'Wheat Bread': Chosen\_Wheat\_Bread, 'White Bread': Chosen\_White\_Bread, 'Oatmeal Cookies': Chosen\_Oatmeal\_Cookies, 'Apple Pie': Chosen\_Apple\_Pie, 'Chocolate Chip Cookies': Chosen\_Chocolate\_Chip\_Cookies, 'Butter,Regular': Chosen\_Butter,Regular, 'Cheddar Cheese': Chosen\_Cheddar\_Cheese, '3.3% Fat,Whole Milk': Chosen\_3.3%\_Fat,Whole\_Milk, '2% Lowfat Milk': Chosen\_2%\_Lowfat\_Milk, 'Skim Milk': Chosen\_Skim\_Milk, 'Poached Eggs': Chosen\_Poached\_Eggs, 'Scrambled Eggs': Chosen\_Scrambled\_Eggs, 'Bologna,Turkey': Chosen\_Bologna,Turkey, 'Frankfurter, Beef': Chosen\_Frankfurter,\_Beef, 'Ham,Sliced,Extralean': Chosen\_Ham,Sliced,Extralean, 'Kielbasa,Prk': Chosen\_Kielbasa,Prk, "Cap'N Crunch": Chosen\_Cap'N\_Crunch, 'Cheerios': Chosen\_Cheerios, "Corn Flks, Kellogg'S": Chosen\_Corn\_Flks,\_Kellogg'S, "Raisin Brn, Kellg'S": Chosen\_Raisin\_Brn,\_Kellg'S, 'Rice Krispies': Chosen\_Rice\_Krispies, 'Special K': Chosen\_Special\_K, 'Oatmeal': Chosen\_Oatmeal, 'Malt-O-Meal,Choc': Chosen\_Malt\_O\_Meal,Choc, 'Pizza W/Pepperoni': Chosen\_Pizza\_W\_Pepperoni, 'Taco': Chosen\_Taco, 'Hamburger W/Toppings': Chosen\_Hamburger\_W\_Toppings, 'Hotdog, Plain': Chosen\_Hotdog,\_Plain, 'Couscous': Chosen\_Couscous, 'White Rice': Chosen\_White\_Rice, 'Macaroni,Ckd': Chosen\_Macaroni,Ckd, 'Peanut Butter': Chosen\_Peanut\_Butter, 'Pork': Chosen\_Pork, 'Sardines in Oil': Chosen\_Sardines\_in\_Oil, 'White Tuna in Water': Chosen\_White\_Tuna\_in\_Water, 'Popcorn,Air-Popped': Chosen\_Popcorn,Air\_Popped, 'Potato Chips,Bbqflvr': Chosen\_Potato\_Chips,Bbqflvr, 'Pretzels': Chosen\_Pretzels, 'Tortilla Chip': Chosen\_Tortilla\_Chip, 'Chicknoodl Soup': Chosen\_Chicknoodl\_Soup, 'Splt Pea&Hamsoup': Chosen\_Splt\_Pea&Hamsoup, 'Vegetbeef Soup': Chosen\_Vegetbeef\_Soup, 'Neweng Clamchwd': Chosen\_Neweng\_Clamchwd, 'Tomato Soup': Chosen\_Tomato\_Soup, 'New E Clamchwd,W/Mlk': Chosen\_New\_E\_Clamchwd,W\_Mlk, 'Crm Mshrm Soup,W/Mlk': Chosen\_Crm\_Mshrm\_Soup,W\_Mlk, 'Beanbacn Soup,W/Watr': Chosen\_Beanbacn\_Soup,W\_Watr}

The solution can be seen below:

|  |  |
| --- | --- |
| **Results** | **Food Units** |
| 52.64371 | Units of Celery |
| 0.25960653 | Units of Frozen Broccoli |
| 63.988506 | Units of Lettuce,Iceberg |
| 2.2929389 | Units of Oranges |
| 0.14184397 | Units of Poached Eggs |
| 13.869322 | Units of Popcorn,Air Popped |
|  |  |
| **Total cost of food** | **$4.34** |

**2. Please add to your model the following constraints (which might require adding more variables) and solve the new model: a. If a food is selected, then a minimum of 1/10 serving must be chosen. (Hint: now you will need two variables for each food i: whether it is chosen, and how much is part of the diet. You’ll also need to write a constraint to link them.)**

Status: Optimal  
Chosen\_Celery,\_Raw = 1.0  
Chosen\_Kielbasa,Prk = 1.0  
Chosen\_Lettuce,Iceberg,Raw = 1.0  
Chosen\_Oranges = 1.0  
Chosen\_Peanut\_Butter = 1.0  
Chosen\_Poached\_Eggs = 1.0  
Chosen\_Popcorn,Air\_Popped = 1.0  
Chosen\_Scrambled\_Eggs = 1.0  
Foods\_Celery,\_Raw = 42.399358  
Foods\_Kielbasa,Prk = 0.1  
Foods\_Lettuce,Iceberg,Raw = 82.802586  
Foods\_Oranges = 3.0771841  
Foods\_Peanut\_Butter = 1.9429716  
Foods\_Poached\_Eggs = 0.1  
Foods\_Popcorn,Air\_Popped = 13.223294  
Foods\_Scrambled\_Eggs = 0.1  
The total food cost with its additional constraints is $4.51

**a. If a food is selected, then a minimum of 1/10 serving must be chosen. (Hint: now you will need two variables for each food i: whether it is chosen, and how much is part of the diet. You’ll also need to write a constraint to link them.)**

for f in foods:   
     diet += foodVars[f] <= 10000000\*chosenVars[f]   
     diet += foodVars[f] >= .1\*chosenVars[f]

*In order to complete this we would need to add two additional pieces to the model:*

*The new variable would consist of:*

yj= 1 if food 𝑗𝑗 is eaten, 0 if not

*In addition to the new variable we would need to add new constraints These can be seen below:*

xj≥0.1 yj for each food jj

0.0000001 xj ≤ yj for each food j

**b. With frozen broccoli and celery being the least favorite we can only accommodate 1 however we can not support both in this model. This leads us to our newest constraint:**

diet += chosenVars['Frozen Broccoli'] + chosenVars['Celery, Raw'] <=1

y frozen broccoli + y raw celery ≤1 (This is due to the fact that they can’t be eaten at the same time)

**c. To get day-to-day variety in protein, at least 3 kinds of meat/poultry/fish/eggs must be selected.**

Now we would need to introduce an even larger constraint to accommodate the following variables.

diet += (chosenVars['Tofu'] + chosenVars['Roasted Chicken'] + chosenVars['Poached Eggs']+chosenVars['Scrambled Eggs']+chosenVars['Bologna,Turkey'] + chosenVars['Frankfurter, Beef']+chosenVars['Ham,Sliced,Extralean'] + chosenVars['Kielbasa,Prk']+chosenVars['Hamburger W/Toppings'] + chosenVars['Hotdog, Plain']+chosenVars['Pork'] +chosenVars['Sardines in Oil'] + chosenVars['White Tuna in Water']) >= 3

|  |  |
| --- | --- |
| **Results** | **Units of Food** |
| 0.1 | units of Bologna,Turkey |
| 42.423026 | units of Celery |
| 82.673927 | units of Lettuce ,Iceberg,Raw |
| 3.0856009 | units of Oranges |
| 1.9590978 | units of Peanut Butter |
| 0.1 | units of Poached\_Eggs |
| 13.214473 | units of Popcorn,Air Popped |
| 0.1 | units of Scrambled\_Eggs |
| Total cost of food | $4.51 |

The results were similar to the original solution however the celery stayed while the broccoli no longer remained. The three proteins available were the bologna, scrambled eggs and porched eggs. We’re able to use the bare minimum of 0.1 units solely based on the fact that the proteins are more-expensive.

**If you want to see what a more full-sized problem would look like, try solving your models for the file diet\_large.xls, which is a low-cholesterol diet model (rather than minimizing cost, the goal is to minimize cholesterol intake). I don’t know anyone who’d want to eat this diet – the optimal solution includes dried chrysanthemum garland, raw beluga whale flipper, freeze-dried parsley, etc. – which shows why it’s necessary to add additional constraints beyond the basic ones we saw in the video! [Note: there are many optimal solutions, all with zero cholesterol, so you might get a different one. It probably won’t be much more appetizing than mine.]**

|  |  |
| --- | --- |
| **Results** | **Units of Food** |
| 0.059863415 | Units of Beans |
| 0.069514608 | adzuki,\_mature\_seeds |
| 0.42866218 | Units of Broccoli raab |
| 0.14694398 | Units of Cocoa mix |
| 0.73805891 | Units of Egg white dried flakes |
| 0.4258564 | Units of Nestle Good with Soy |
| 0.050114149 | Units of Ross ISOMOL |
| 0.15033656 | Units of Margarine like spread |
| 0.25918767 | Units of Mung beans mature seeds raw |
| 0.18052856 | Units of Nuts mixed nuts dry roasted |
| 1.184482 | Units of Oil vegetable sunflower linoleic |
| 0.10375187 | Units of Seeds sunflower seed kernels |
| 0.031866196 | Units of Snacks potato chips |
| 0.070710308 | Units of Spices paprika |
| 0.55106575 | Units of Tomatoes sun dried |
| 9999.6864 | Units of Water bottled non carbonated,\_CALISTOGA |
| Total cholesterol | 0 |

The solution to the diet problem when maximizing the protein in the diet as opposed to minimizing the cholesterol can be seen below:

|  |  |
| --- | --- |
| **Results** | **Units of Food** |
| 7.0117007 | Units of BANQUET\_Salisbury\_Steak\_Meal |
| 0.20365743 | Units of Cereals\_ready\_to\_eat,\_KASHI\_Heart\_to\_Heart\_by\_KELLOGG |
| 0.23412086 | Units of Collards,\_raw |
| 25.855235 | Units of Fish,\_devilfish,\_meat\_(Alaska\_Native) |
| 31.46708 | Units of Fish,\_lingcod |
| 0.02 | Units of Fish\_oil,\_cod\_liver |
| 2.2140307 | Units of Gelatins,\_dry\_powder,\_unsweetened |
| 0.037489833 | Units of Mollusks,\_oyster,\_eastern,\_canned |
| 57.437865 | Units of Rhubarb,\_wild,\_leaves\_(Alaska\_Native) |
| 621.79859 | Units of Sweeteners,\_tabletop,\_aspartame,\_EQUAL,\_packets |
| 9.5089609 | Units of Tea,\_brewed,\_prepared\_with\_distilled\_water # |
| 9552.2849 | Units of Water,\_bottled,\_non\_carbonated,\_CALISTOGA |
| 276.5536 | Units of Water,\_bottled,\_non\_carbonated,\_DANNON |
| 0.076732592 | Units of Whale,\_beluga,\_flipper,\_raw\_(Alaska\_Native) |
| 9.6405544 | Units of Whale,\_beluga,\_liver,\_raw\_(Alaska\_Native) |
| 1.7353546 | Units of Whale beluga,\_meat,\_air\_dried,\_raw\_(Alaska\_Native) |
| **2994.899576** | **Total protein** |